Information and the Cost of Capital

David Easley
Department of Economics
Cornell University

and

Maureen O'Hara
Johnson Graduate School of Management
Cornell University

February 2001

*We would like to thank Christopher Gadarowski, Jerry Hass, Soeren Hvidkjaer, Eugene Kandel, Wayne Ferson, Rene Stulz and seminar participants at Columbia, Cornell and Michigan State for helpful comments. The authors can be reached at dae3@cornell.edu and mo19@cornell.edu.

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1. Introduction

Fundamental to a variety of corporate decisions is a firm's cost of capital. From determining the hurdle rate for investment projects to influencing the composition of the firm's capital structure, the cost of capital influences the operations of the firm and its subsequent profitability. Given this importance, it is not surprising that a wide range of policy prescriptions have been advanced to help companies lower this cost. For example, Arthur Levitt, the outgoing chairman of the Securities and Exchange Commission, suggests that "high quality accounting standards …improve liquidity [and] reduce capital costs".[1] The Nasdaq stock market argues that its trading system "most effectively enhances the attractiveness of a company's stock to investors".[2] And investment banks routinely solicit underwriting business by arguing that their financial analysts will lower a company's cost of capital by attracting greater institutional following to the stock. While accounting standards, market microstructure, and financial analysts each clearly differ, these factors all can be thought of as influencing the information structure surrounding a company's stock.[3]

Paradoxically, asset-pricing models include none of these factors in determining the required return for a company's stock. While more recent asset-pricing models (see Fama and French [1992; 1993]) admit the possibility that something other than market risk may affect

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[1] Remarks by Arthur Levitt, Inter-American Development Bank, September 29, 1997. Also cited in Admati and Pfleiderer (2000).
[2] See the Nasdaq website, www.nasdaq.com.
[3] Indeed, Arthur Levitt argues even further: "Quality information is the lifeblood of strong, vibrant markets. Without it, liquidity dries up. Fair and efficient markets cease to exist." See remarks at Economic Club of Washington, April 6, 2000.

required returns, these alternative factors do not include the role of information. This exclusion is particularly puzzling given the presumed importance of market efficiency in asset pricing. If information matters for the market, why then should it not also matter for the firms that are in it?

In this research we investigate the role of information in affecting a firm's cost of capital. Our particular focus is on the specific roles played by public and private information. The argument we develop here is that differences in the composition of information between public and private information affect the cost of capital, with investors demanding a higher return to hold stocks with greater private information. This higher return reflects the fact that private information increases the risk to uninformed investors of holding the stock because informed investors are better able to shift their portfolio weights to incorporate new information. This cross-sectional effect results in the uninformed traders always holding too much of stocks with bad news, and too little of stocks with good news. Holding more stocks cannot remove this risk because the uninformed are always on the wrong side; holding no stocks is sub-optimal because uninformed utility is higher holding some risky assets. Moreover, the standard separation theorem that typically characterizes asset pricing models does not hold here because there is no market portfolio; informed and uninformed investors hold different portfolios because they perceive different risks and returns.[4] Private information thus induces a new form of systematic risk, and in equilibrium investors require compensation for this risk.

We develop our results in a multi-asset rational expectations equilibrium model that includes public and private information, and informed and uninformed investors. Important features of the model are risk averse investors, a positive net supply (on average) of each risky

[4] The assets in the market, of course, still exist, but the informed traders will hold different weights of each asset in their portfolios depending on the information they learn. The uninformed don't know the information, so they are unable to replicate these optimal weights, and will end up holding a different portfolio than that of the informed traders.

asset, and incomplete markets. We find a non-revealing rational expectations equilibrium in which assets generally command a risk premium. The model demonstrates how in equilibrium the quantity and quality of information affects asset prices, resulting in cross-sectional differences in firms' required returns. What is particularly intriguing about the model is that it demonstrates a role for both public and private information to affect a firm's required return. This provides a rationale for how an individual firm can influence its cost of capital by choosing features like its accounting treatments, financial analyst coverage, and market microstructure. We also show why firms with little available information, such as IPOs, face high costs of capital: in general, more information, even if it is privately held, is better than no information at all.

Prior researchers have investigated how private information affects asset prices in a variety of contexts. Three streams of the literature are most relevant for our work here. First, building from the classic analysis by Grossman and Stiglitz [1980], a number of authors have looked at the role of private information in rational expectations models. Admati [1985] analyzed the effects of asymmetric information in a multi-asset model. Her analysis focused on how an asset's equilibrium price is affected by information on its own fundamentals and those of other assets. Because agents in her model have diverse information, she finds that each agent has a different risk-return trade-off; a result very similar to our finding here that there is no market portfolio. While Admati provides an elegant analysis of multi-asset equilibrium, her focus is not on the public versus private information issues we consider. Wang [1991] showed in a two-asset multi-period model that asymmetric information induces two effects into asset prices. First, uninformed investors require a risk premium to compensate them for the adverse selection problem that arises from trading with informed traders. Second, informed trading also makes prices more informative, thereby reducing the risk for the uninformed and lowering the risk premium. The overall effect on the equilibrium required return in this model is ambiguous. Because the model allows only one risky asset, it is not clear how, if at all, information affects cross-sectional returns, or how information affects portfolio selection.[5] One way to interpret our results is that holding the amount of information constant, the adverse selection effect prevails, so that in our multi-asset equilibrium cross-sectional effects arise.

Dow and Gorton [1995] provide an alternative analysis in which informed traders profit from their information, and consequently uninformed traders lose relative to the informed. For profitable informed trade to be possible the equilibrium must not be fully revealing and the uninformed must not be able to hold the market portfolio. We do this with the standard device of noise trade so that we can focus on the effect of private versus public information on the cost of capital. Dow and Gorton do not have noise traders and instead restrict the uninformeds' portfolios so that they cannot buy the market. They do not consider public versus private information or the cost of capital, but their approach could also be used to address these issues.

A second stream of related research considers the role of information when it is incomplete but not asymmetric.[6] Of particular relevance here is Merton [1987] who investigates the capital market equilibrium when agents are unaware of the existence of certain assets. In Merton's model, all agents who know of an asset agree on its return distribution, but information is incomplete in the sense that not all agents know about every asset. Merton shows that in equilibrium the value of a firm is always lower with incomplete information and a smaller investor base. In our model, all investors know about every asset but information is asymmetric:

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[5] Another important difference between our work and that of Wang[1991] or Admati [1985] is that these authors do not consider the role of public information. As we show here, increasing the quantity of public information about an asset can affect the asset's equilibrium required return.

[6] Yet another stream of research in this area considers the effects of uncertainty about and estimation of return distribution parameters. This estimation risk raises the required return for investors, (see Barry and Brown [1984];

some investors know more than others about returns. While both approaches lead to cross-sectional differences in the cost of capital, we show that there is an important difference with respect to their robustness to arbitrage.

Finally, a third stream of related research considers the role of information disclosure by firms. Disclosure essentially turns private information into public information, so this literature addresses the role of public information in affecting asset prices. Diamond [1985] developed an equilibrium model in which public information makes all traders better off. What drives this result is that information production is costly, and so disclosure by the firm obviates the need for each individual to expend resources on information gathering.[7] While our model also shows a positive role for public information, our result arises because public information reduces the risk to uninformed traders of holding the asset. Diamond and Verrecchia [1991] consider a different risk issue by analyzing how disclosure affects the willingness of market makers to provide liquidity for a stock. Using a Kyle [1985]-type model, they show that disclosure changes the risks to market makers, which in turn induces entry or exit by dealers. In this model, disclosure can improve or worsen liquidity depending upon these dealer decisions. Our analysis does not consider dealers, but the models are related in that public information influences the riskiness of holding the stock. Research by Fishman and Haggerty [1995] and Admati and Pfleiderer [2000] considers other important aspects of disclosure, such as the role of insiders and strategic issues in disclosure, but these issues are outside of the scope of the problem considered here.[8]

Coles and Lowenstein [1988]; and Coles, Lowenstein and Suay [1995]). See also Basak and Cuoco [1998] and Shaapiro [2000] for more development of the investor recognition hypothesis.

[7] Diamond [1985] notes that in his model "an even better arrangement would be an agreement among traders to all refrain from acquiring any new information, or a tax on information acquisition". In our model, more information is always better than less information so that this effect does not arise. We do not consider the role of information cost in our model, so our equilibrium is not affected by the cost issues investigated by Diamond.

[8] Fishman and Haggerty [1995] investigate how disclosure affects the utility of corporate insiders and outsiders. They find that mandatory disclosure can make insiders better off, even when insiders do not actually know any value-relevant information. Admati and Pfleiderer [2000] provide a very interesting analysis of the externality that

What emerges from our research is a demonstration of why a firm's information structure affects its return in equilibrium. This dictates that a firm's cost of capital is also influenced by information, providing an important linkage between asset pricing, corporate finance, and the information structure of corporate securities. A particular empirical prediction of our model is that in comparing two stocks that are otherwise identical, the stock with more private information and less public information will have a larger expected excess return. In a companion empirical paper (Easley, Hvidjkaer, and O'Hara [2000]) we test this prediction using a structural microstructure model to provide estimates of information-based trading for a large cross section of stocks. Our findings there provide strong evidence of the effects we derive here. Our model also develops a number of other empirical implications, as yet untested, on the effects of the dispersion of information of information, of the quantity of information, and of the quality of public and private information on a firm's cost of capital.

This paper is organized as follows. The next section develops a rational expectations model including many assets, many sources of uncertainty, and informed and uninformed traders. We characterize the demands of the informed and uninformed traders, and we demonstrate that a non-revealing rational expectations equilibrium exists. In Section 3, we then analyze the equilibrium and determine how the equilibrium return differs across stocks. In this section, we derive our results on the specific influence of private and public information on asset returns. Section 4 then considers the impact of various aspects of a firm's information structure on its cost of capital. Section 5 discusses some extensions and generalizations of our model. Section 6 is a conclusion.

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disclosure imposes on firms. Because one firm's disclosure may be informative for other firms, it is cheaper for a firm to let others do costly disclosure. Without required disclosure, therefore, there may be an under-provision

2. Information and Asset Prices in Equilibrium

In this section, we develop a rational expectations equilibrium model in which both public and private information can affect asset values. We first describe the information surrounding a company's securities, and how this information is disseminated to traders. We then derive demands for each asset by informed traders who know the private information and by uninformed traders who do not. Because informed traders' information affects their demands it is reflected in equilibrium prices. In a rational expectations equilibrium, uninformed traders make correct inferences about this private information from prices. We solve for rational expectations equilibrium prices and derive the equilibrium required return for each asset. This required return is the company's cost of capital.

2.1 The Basic Structure

We consider a two-period model: today when investors choose portfolios and tomorrow when the assets in these portfolios pay off. There is one risk-free asset, money, which has a constant price of *1*. There are K risky stocks indexed by $k = 1, \ldots, K$. Future values, v_k, are independently, normally distributed with mean $\bar{v}_k$ and precision $\boldsymbol{r}_k$.[9] The per capita supply of stock k, $\overline{x_k}$, is also normally distributed with mean $\bar{x}_k$ and precision $\boldsymbol{h}_k$.[10] Stock prices, p_k, are

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of public information.

[9] Including correlation in future values would complicate our analysis, but not change our conclusions. Market risk would be priced as usual. The remaining risk that we focus on here would then be priced as it is in our analysis.

[10] Assuming random net supply is a standard modeling device in rational expectations models. One theoretical interpretation is that this approximates noise trading in the market. A more practical example of this concept is the current switch for portfolio managers toward using float-based indices from shares-outstanding indices. This shift is occurring because for many stocks, the actual number of shares that trades in the market is a more meaningful number than is the number of shares that exist. Determining this "float" essentially means finding the distribution of shares that trade in a given stock (a random variable), rather than taking the supply as given.

determined in the market. Traders trade today at prices $\left(1, p_1, \ldots, p_k\right)$ per share and receive payoffs tomorrow of $\left(1, \tilde{v}_1, \ldots, \tilde{v}_k\right)$ per share.

Investors receive signals today about the future values of these stocks. For stock k, I_k signals, s_{ki}, are drawn independently from a normal distribution with mean v_k, the future value of stock k, and precision g_k. Some of these signals are public and some are private. The fraction of the signals about the value of stock k that are private is denoted a_k; the fraction of the signals that are public is $1 - a_k$. Public signals are received by all investors before trade begins. Private signals are received only by informed traders. We let m_k be the fraction of traders who receive the private signals about stock k. All of these random variables are independent and their distributions are known to the investors.[11]

There are $K + 1$ assets, and so K relative prices, and many sources of uncertainty: signals about the future value of the stocks and the random supply of each stock. We view the random supply of stocks as a simple proxy for noise trade, but it is important. Without the high dimensional information space there would be a fully revealing rational expectations equilibrium in which the uninformed investors could completely infer the informed investors' information

[11] More precisely, the signals s_{ki} are independent conditional on v_k.

from equilibrium asset prices. It would then not matter whether information was public or private.

There are J investors indexed by $j = 1, \ldots, J$. These investors all have CARA utility with coefficient of risk aversion $d>0$. These investors must in equilibrium hold the available supply of money and stocks. Markets are incomplete, so stocks are risky even for informed investors. Because the investors are risk averse, and the stocks are risky, the risk will be priced in equilibrium. The question that we are interested in is how the distribution of information affects asset prices and thus expected returns.

2.2 Investors' Decision Problems

Each investor chooses his demands for assets $k = 1, \ldots, K$ to maximize his expected utility subject to his budget constraint. The budget constraint today for typical investor j is $m^j + \sum_k p_k z_k^j = \overline{m^j}$, where z_k^j is the number of shares of stock k he purchases, m^j is the amount of money he holds and $\overline{m^j}$ is his initial wealth. His wealth tomorrow is the random variable $\tilde{w}^j = \sum_k v_k z_k^j + m^j$. Substituting from the budget constraint for $\overline{m^j}$, investor j's wealth can be written as $\tilde{w}^j = \sum_k (v_k - p_k) z_k^j + \overline{m}^j$.

Suppose that conditional on all of investor j's information, he conjectures that the distribution of v_k is normal with mean $\bar{v}_k^j$ and precision r_k^j. Then, because he has CARA utility and all distributions are normal, investor j's objective function has a standard mean-variance maximization expression

$$\underset{\left(z_k^j\right)_{k=1}^K}{Max}\; E\left[\tilde{w}^j\right] - (\boldsymbol{d}/2)\, Var^j\left[\tilde{w}^j\right] \tag{1}$$

or equivalently

$$\underset{\left(z_k^j\right)_{k=1}^K}{Max} \sum_k \left(\tilde{v}_k^j - p_k\right) z_k^j + \bar{m}^j - \boldsymbol{d}/2 \sum_{k=1}^K \left[\left(\boldsymbol{r}_k^j\right)^{-1}\left(z_k^j\right)^2\right]. \tag{2}$$

So investor j's demand function for asset *k* is

$$z_k^j = \frac{\overline{v_k^j} - p_k}{\boldsymbol{d}\left(\boldsymbol{r}_k^j\right)^{-1}}. \tag{3}$$

The demand function for asset *k* in (3) depends upon investor *j*'s beliefs about the asset's risk and return. These beliefs differ depending upon whether the agent is informed or not. We first consider these beliefs for informed investors. It follows from Bayes Rule that if *j* is informed, then his predicted distribution for v_k is Normal with conditional mean

$$\overline{v_k^j} = \frac{\boldsymbol{r}_k \overline{v_k} + \boldsymbol{g}_k \sum_{i=1}^{I_K} s_{ki}}{\boldsymbol{r}_k + \boldsymbol{g}_k I_K} \tag{4}$$

and conditional precision

$$\boldsymbol{r}_K^i = \boldsymbol{r}_K + \boldsymbol{g}_K I_K \tag{5}$$

Thus, from (3) the demand for asset *k* by informed investor *j* is

$$z_k^{j*} = \frac{r_k \overline{v}_k + g_k \sum_{i=1}^{I_k} s_{ki} - p_k \left(r_k + g_k I_k \right)}{d} \equiv DI_k^* \left(\sum_{i=1}^{I_k} s_{ki}, p_k \right) \tag{6}$$

Solving for uninformed investors' demands is more complicated. These investors know the public signals, but not the private signals. What they do know, however, is that the demands of the informed traders affect the equilibrium price, and so they rationally make inferences about the underlying information from the price. To learn from the price, these investors must conjecture a form for the price function, and in equilibrium this conjecture must be correct. Suppose the uninformed conjecture the following price function

$$p_k = a\overline{v}_k + b \sum_{i=1}^{aI_k} s_{ki} + c \sum_{i=a_k I_{k+1}}^{I_k} s_{ki} - dx_k + e\overline{x}_k \tag{7}$$

To compute the distribution of v_k, conditional on p_k, it is convenient to define the random variable q_k to be

$$q_k = \frac{p_k - a\overline{v}_k - c \sum_{i=a_k I_{k+1}}^{I_k} s_{ki} + \overline{x}_k (d - e)}{b a_k I_k} = \frac{\sum_{i=1}^{a_k I_k} s_{ki}}{a_k I_k} - \left(\frac{d}{b a_k I_k} \right) \left(x_k - \overline{x}_k \right) \tag{8}$$

What is important for our purposes is that the uninformed investors can compute q_k and that q_k has mean v_k. Calculation shows that q_k is normally distributed with mean v_k and precision r_{qk} where

$$r_{qk} = \left[\left(\frac{d}{b a_k I_k} \right)^2 h_k^{-1} + \left(\frac{1}{a_k I_k} \right) g_k^{-1} \right]^{-1}. \tag{9}$$

Using this information, we can compute the conditional mean and variance from the perspective of the uninformed trader. These are

$$\overline{v_k^j} = \frac{r_k \overline{v_k} + g_k \sum_{i=a_k I_{k+1}}^{I_k} s_{ki} + r_{qk} q_k}{r_k + g_k (1 - a_k) I_k + r_{qk}} \tag{10}$$

and

$$r_k^j = r_k + g_k (1 - a_k) I_k + r_{qk} \,. \tag{11}$$

Each uninformed trader's demand for asset *k* is thus

$$z_k^{j*} = \frac{r_k \overline{v_k} + g_k \sum_{i=a_k I_k + 1}^{I_k} s_{ki} + r_{qk} q_k - p_k \left(r_k + g_k (1 - a_k) I_k + r_{qk} \right)}{d} \equiv DU_k^* \left(\sum_{i=a_k I_k + 1} s_{ki}, q_k, p_k \right) \tag{12}$$

In the next section, we show that there is a rational expectations equilibrium in which the conjectures used to compute these demands are correct.

2.3 Equilibrium

In equilibrium, for each asset *k*, per capita supply must equal per capita demand or

$$m_k DI_k^* \left(\sum_{i-1}^{I_k} s_{ki}, p_k \right) + (1 - m) DU_k^* \left(\sum_{i=a_k I_k + 1}^{I_k} s_{ki}, q_k, p_k \right) = x_k \tag{13}$$

We find the equilibrium by solving equation (13) for p_k and then verifying that p_k is of the form conjectured in (7). Proposition 1 characterizes this equilibrium.

Proposition 1: There exists a non-revealing rational expectations equilibrium in which, for each asset *k*,

$$p_k = a\overline{v_k} + b\sum_{i=1}^{a_k I_k} s_{ki} + c\sum_{i=a_k I_k+1}^{I_k} s_{ki} - dx_k + e\overline{x}_k$$

where $$a = {r_v}/{C_k}, \quad b = \frac{m_k g_k + \dfrac{(1-m_k)\, r_{qk}}{a_k I_k}}{C_k}, \quad c = {g_k}/{C_k},$$

$$d = \frac{d + \dfrac{(1-m_k)\, r_{qk} d}{a_k I_k m_k g_k}}{C_k}, \text{ and } e = \frac{\dfrac{(1-m_k)\, r_{qk} d}{a_k I_k m_k g_k}}{C_k}, \text{ where}$$

$$C_k = r_k + (1-a_k)\, I g_k + m_k a_k I_k g_k + (1-m_k)\, r_{qk}, \text{ and}$$

$$r_{qk} = \left[(m_k g_k a_k I_k)^{-2} h_k^{-1} d^2 + (a_k I_k g_k)^{-1} \right]^{-1}.$$

Proof: See Appendix.

The proposition demonstrates that there exists a rational expectations equilibrium in which prices are non-revealing. An interesting feature of this equilibrium is that the portfolios of risky assets held by informed and uninformed traders differ. In a standard CAPM framework, investors reach the efficient frontier by selecting the mix of risk-free bonds and the "market" portfolio that maximizes their utility. Here, there is no market portfolio. With asymmetric information, the expected risk and return from holding any security *k* differs depending upon the traders' access to private information, and so, too, will the portfolio of risky securities they hold. The market portfolio will thus be different for informed and uninformed investors.

3. Information and Cross-Sectional Asset Returns

Having established the equilibrium, we now turn in this section to an analysis of how the equilibrium return differs across stocks. We show that this return depends on the information structure, with the levels of public and private information influencing the cross-sectional equilibrium return demanded by investors. The random return per share to holding asset k is v_k -p_k. The expected, or average, return on asset k is thus $E[v_k - p_k]$, where the expectation is computed with respect to prior information. The following proposition describes the equilibrium risk premium on asset k.

Proposition 2: The expected return on stock k is given by

$$E[v_k - p_k] = \frac{d\bar{x}_k}{r_k + (1 - a_k) I_k g_k + m_k a_k I_k g_k + (1 - m_k)\, r_{qk}} .$$

Proof: See Appendix

Proposition 2 reveals a number of important properties of equilibrium asset returns. Inspecting the numerator reveals that the risk premium of a stock depends on agents risk preferences (d) and on the per capita supply $\bar{x}_k$ of the stock. Obviously, if agents are risk neutral $(d=0)$, then the asset's underlying risk is not important to them, negating the need for any risk premium. If agents are risk averse, then there is positive risk premium for asset k as long as the per capita supply of asset is on average positive. It is important to note that $\bar{x}_k$ is the per capita number of shares of asset k and not the portfolio weight, or fraction of wealth invested in asset k. In an economy with a large number of assets the portfolio weights for all assets are small, but this has no effect on the expected return given in Proposition 2. Of course, if $\bar{x}_k = 0$ for all stocks then there is no risk premium for any stock. In such a world there is on average no per capita supply of any asset that needs to be held, so no agent has to bear any risk, and risk bearing is thus not rewarded. Even market risk would not be priced in this uninteresting economy. We focus instead on economies with assets that are in positive per capita supply and which thus have positive expected return.

The risk premium is also affected by the stock's information structure. The denominator shows the influence of traders' prior beliefs and the effects of public and private information. If information on the asset is perfect (perfect prior information, $r_v = \infty$, or perfect signals, $g_v = \infty$), then asset k is risk free and its price is its expected future value. When information is not perfect the risk premium is positive. The greater the uncertainty about the asset's value, the smaller is the precision, and the greater is the stock's risk premium. In the following analysis, we examine the economically interesting case in which $d > 0$, $\overline{x_k} > 0$, $r_v < \infty$, $g_k < \infty$, and thus there is a positive expected return on stock k.

We are interested in cross-sectional variation in this return. Most important is how the required return is affected by the amount of private information versus public information, i.e., a_k. Proposition 3 details this effect.[12]

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[12] We treat a_k and I_k as continuous variables, ignoring integer constraints on $a_k I_k$ and $(1-a_k)I_k$. Equivalent results could be obtained by changing $a_k I_k$ and $(1-a_k)I_k$ in integer units.

Proposition 3: For any stock k, and provided $m_k < 1$, shifting information from public to private increases the equilibrium required return, or

$$\frac{\partial E[\tilde{v} - p_k]}{\partial a_k} > 0$$

Proof: See Appendix

The proposition shows that if private signals are truly private to some traders $(m_k < 1)$ then the required return is increasing in a_k, the fraction of the signals about stock k that are private (when $m_k = 1$, all available information is actually public). This result has an important implication for cross-sectional returns: in comparing two stocks that are otherwise identical, the stock with more private and less public information will have a larger expected excess return. This occurs because when information is private, rather than public, uninformed investors cannot perfectly infer the information from prices, and consequently they view the stock as being riskier. Cross-sectional returns on stocks will thus depend on the structure of information in each individual stock.

One might conjecture that this effect would be removed by the uninformed investors optimally diversifying, or by simply not holding stocks with a large amount of private information. But this is not the case. Uninformed investors chose not to avoid this risk in equilibrium. They are rational so they hold optimally diversified portfolios, but no matter how they diversify they lose relative to the informed traders. To completely avoid this risk, the uninformed traders would have to hold only money, but this is not optimal; their utility is higher by holding the risky stocks. Although the model has only one trading period it is easy to see that uninformed investors also would not choose to avoid this risk by buying and holding a fixed portfolio over time. In each trading period in an inter-temporal model uninformed investors reevaluate their portfolios. As prices change, they optimally change their holdings.

Could the uninformed arbitrage this effect away (or conversely, make arbitrage profits) by simply holding all high α stocks and shorting all low α stocks? Again, the answer is no. It is true that everyone, including the uninformed, know the α's. But the uninformed do not know the actual private information. Holding all high α stocks is extremely risky for the uninformed because these stocks have both good and bad news. The informed are able to buy more of the good news stocks, and hold less of (or even short) the bad news stocks, thereby allowing them to exploit their information; this option is not available to the uninformed. This property of our equilibrium highlights an important difference between our asymmetric information model and Merton's [1987] incomplete information model. In Merton's model, arbitrage is possible, "if such stocks can be easily identified and if accurate estimation of the alphas (the excess returns) can be acquired at low cost…then professional money managers could improve performance by following a mechanical investment strategy titled towards these stocks. If a sufficient quantity of such investments were undertaken then this extra excess return would disappear".

In our model, everyone knows about the stocks, but they do not know what position to take. This difficulty is related to our earlier observation that there is no single market portfolio. The informed hold different weights of the assets in their portfolio than do the uninformed. The uninformed cannot mimic the informed portfolio by holding all good information stocks because they do not know the information value, and holding equal weights of all of the stocks does not remove this risk.

The intuition for this result is similar to that of Rock's IPO [1986] underpricing explanation. In that analysis, the uninformed bid for new issues and so do informed insiders. When the information is good, the insiders buy larger amounts, and the uninformed correspondingly get less. When the information is bad, the insiders do not buy the new issue, and the uninformed end up holding most of it. Because the uninformed know this will happen to them, in equilibrium they demand a higher expected return to compensate. Here, the problem extends across all the assets in that private information will again influence the portfolio outcomes of the informed and uninformed. Our result is that equilibrium asset returns will reflect this risk.

The differing information that traders have results in differing perceptions of the efficient mean-standard deviation (of wealth) frontier. This causes informed and uninformed traders to select different portfolios. The perceived efficient frontier is linear, with a slope determined the trader's perception of mean returns $\left(\bar{v}_k^j\right)$ and standard deviations $\left(\left(\boldsymbol{r}_k^j\right)^{-1/2}\right)$ for assets. For an economy with one risky asset (and one riskless asset) the slope of this frontier according to trader j is $\left(\bar{v}_1^j - p_1\right)\left(\boldsymbol{r}_1^j\right)^{-1/2}$. The trader's indifference curves in expected wealth $\left(\bar{w}\right)$—standard deviation of wealth (s_w) space have slope $d\bar{v}_w$. The portfolio choice problem for trader *j* is represented graphically by Figure 1.[13]

If there is any private information about the risky asset, then in equilibrium informed traders have a larger precision; for informed trader I and uninformed trader U, respectively. The expected value of the asset depends on the information that informed traders receive, and uninformed traders partially infer from price. On average, these expected values are both equal to the prior expected value of the asset, $E\left[\bar{v}^I\right] = E\left[\bar{v}^U\right] = \bar{v}$. Figure 2 shows the average portfolio choices of informed and uninformed traders denoted by X^I and X^U.

On average, informed traders take on more risk by holding more of the risky asset. Informed traders' beliefs about mean return are more responsive to signals than are uninformed traders' beliefs. So when there is good news, the informed hold even more of the risky asset, and when there is bad news their holdings are reduced by more than are the uninformed traders' holdings. If the news is bad enough, the informed hold less of the risky asset than do the uninformed. This effect is captured in Figure 3, where I_G and I_B (U_G and U_B) are the efficient frontiers for informed (uninformed) traders given Good news and Bad news, respectively.

Another way to see how this effect works is to compute the equilibrium portfolios of informed and uninformed investors. Let Z_k^U be the per capita demand for stock k by uninformed traders, and let Z_k^I be the per capita demand for stock k by informed traders. Stocks are riskier for uninformed traders than they are for informed traders, and so one might expect this to affect

[13] The same analysis holds for multiple assets as long as markets are incomplete.

how much of any stock they hold. To determine this, we first calculate the difference in holdings of asset *k* by the informed and uninformed

$$Z^I - Z^U = \left[\left(\sum_{i=1}^{a_k I_k} s_i\right)\left(g_s - \frac{r_{qk}}{a_k I_k}\right) + p_k\left[r_{qk} - a_k I_k\right] + r_{qk}\left(\frac{1}{m_k g_k a_k I_k}\right)(x_k - \bar{x}_k)\right]d^{-1} \qquad (14)$$

It is easy to see that $Z_k^I - Z_k^U$ is normally distributed with a strictly positive mean. Using this fact, we can then calculate the difference in average holdings of asset *k* by the informed and uninformed, or

$$E\left[Z_k^I - Z_k^U\right] = d^{-1} E\left[\tilde{v}_k - p_k\right](a_k I_k g_k - r_{qk}) > 0 \qquad (15)$$

The positive sign in equation (15) dictates that the informed investors are holding on average more of each risky asset *k* than are the uninformed investors. How much more is seen to depend on three factors: the risk aversion coefficient, the expected return, and the difference in precisions of the informed and uninformed traders' information.

An interesting question is how does realized private information affect stock holdings? Earlier we argued that informed traders are able to capitalize on their information by shifting their portfolios relative to those of the uninformed. This private news is captured by the sum of the private signals, $\sum_{i=1}^{a_k I_k} s_i$, with good news raising this value and bad news lowering it. We can determine how private news affects the actual portfolios of informed and uninformed investors by calculating

$$\frac{\partial\left(Z_k^I - Z_k^U\right)}{\partial \sum_{i=1}^{a_k I_k} s_i} > 0 \tag{16}$$

So good private information raises the informed's holding of asset k relative to the uninformed, while bad private news has the opposite effect. Thus, while on average the informed hold more of the risky asset k than do the uninformed, their actual holding in any period will be more or less than the uninformed's holding depending upon their specific private information.

How does the value of public information affect these portfolios? Because all traders see the public news, one might conjecture that it has no effect, but this is incorrect. To see why, note that the public information is $\sum_{i=a_k I_k+1}^{I_k} s_i$. Again, positive public news raises this value, and negative public news lowers it. Computing the impact of public news on the holdings of the informed and uninformed, we find

$$\frac{\partial\left(Z_k^I - Z_k^U\right)}{\partial \sum_{i=a_k I_k+1}^{I_k} s_i} < 0 \tag{17}$$

Thus, good public information lowers the holdings of asset k by informed traders relative to the uninformed holdings. The reason this occurs is that, from the perspective of the uninformed trader, asset k is less risky when there is good public news. This induces the uninformed to hold more of the asset, which closes the gap between the informed and uninformed holdings.

The portfolio changes induced by public and private news demonstrate the channel by which information affects cross-sectional asset returns. In the next section, we investigate this linkage in more detail by looking at the role played by the characteristics of public and private information.

4. Information and the Cost of Capital

Our analysis thus far reveals that greater private information increases the return investors require to hold that stock in equilibrium. Viewing this result from the perspective of the firm, a firm whose stock has relatively more private information thus faces a higher cost of equity capital. We now turn to understanding the factors that increase or decrease this cost of capital.

In our model, the dispersion of private information is captured by the variable m_k, the fraction of traders who receive the private information. A higher value of m_k, means that more traders know the information, and in equilibrium this influences the risk premium of the stock through two channels. First, the stock is less risky for informed traders than it is for uninformed traders. Thus, on average, informed traders hold greater amounts of the stock. So if more traders are informed then, on average, demand for the stock increases, the price increases, and the firm's cost of capital falls. Second, there is an indirect effect on the cost of capital through the revelation of information by the stock price. If more traders are informed, then their information is revealed with greater precision to the uninformed. This makes the stock less risky for the uninformed and this further reduces the cost of capital.

These risk premium effects are captured by the comparative static result

$$\frac{\partial E(\tilde{v}_k - \tilde{p}_k)}{\partial m_k} < 0 . \qquad (18)$$

This finding demonstrates that a greater dispersion of private information actually lowers the required risk premium, and thus lowers a company's cost of capital.

This result highlights the complex role that information plays in equilibrium. While the informed benefit from knowing private information, they also must contend with the fact that their own trades impound this information into the stock price. The more informed agents there are, the more informative are their collective trades, and the more information is reflected in the equilibrium price. If all agents become informed, then as discussed in Proposition 3, all information is essentially public and there is no risk premium for private information.

Taken together, our results on the existence and dispersion of information suggest that firms could lower their cost of capital by either reducing the extent of private information or by increasing its dispersion across traders. There are several potential ways of doing so. For example, firms could disclose information to the market that would otherwise be privately known. The optimal amount of disclosure by firms has been investigated by numerous authors in numerous contexts but our analysis here shows why this lowers the cost of capital: substituting public for private information lowers the risk premium investors demand in equilibrium. Botosan [1997] provides empirical evidence on this effect by showing that for a sample of firms with low analyst following, greater disclosure reduces the cost of capital by an average of 28 basis points.

It may be, however, that firms do not know the underlying private information, and so are unable to disclose it to the market. Alternatively, even if they do know it, the moral hazard problems of self-reporting information may lead the market to be dubious of any such disclosures. But firms can encourage greater scrutiny of the company by financial analysts, who may aid in the both the development and dissemination of information.[14] It is also in the company's best interest to increase the quality of the information on the firm. Returning to the risk premium equation (13), it is straightforward to show that the precision of both public and private information affects the required return, or

[14] Whether analysts actually uncover new information, or simply disseminate what is already known to at least some traders is a subject of debate. Note, however, that in our model just disseminating information would increase μ_k, and this would lower the firm's cost of capital. For a discussion of information and analysts, see Easley, O'Hara and Paperman [1998].

$$\frac{\partial E(\tilde{v}_k - p_k)}{\partial \boldsymbol{r}_s} < 0 \tag{19}$$

This finding reinforces the role played by analysts in affecting asset returns. The forecast of any one analyst may have low precision, but the collective forecast of many analysts should be much more accurate. Thus, companies benefit from having many analysts because analysts increase the precision of information and this lowers the companies' cost of capital.

These findings suggest an important role for the accuracy of accounting information in asset pricing. Here greater precision will directly lower a company's cost of capital because it will reduce the riskiness of the asset to the uninformed. This finding is consistent with the extensive accounting literature documenting the effects of accounting treatments on stock prices. Given that accounting changes do not affect the company's underlying business or economic profits, standard asset pricing models would not suggest any impact on stock prices. Our model demonstrates why this reasoning is wrong; because information affects asset prices, the quantity and quality of that information is very relevant for asset price behavior.

An interesting feature of our model is that the life-cycle of a firm may also influence its cost of capital. In particular, it seems reasonable that a firm with a long operating history will be better known by investors. This is captured in our model by the prior belief, in that investors will have a greater prior precision if they know more about the firm. In our model, the precision of the prior belief has a direct effect on the risk premium given by

$$\frac{\partial E(\tilde{v}_k - p_k)}{\partial \boldsymbol{r}_k} < 0 \tag{20}$$

Thus, the greater the prior precision, the lower the cost of capital. This finding is consistent with the oft-observed regularity that more established firms find it easier, and cheaper, to raise funds in the market.

This finding is also consistent with the empirical results of Coval and Moskowitz [1999] and Huberman [2000] who find that money managers and investors are more comfortable holding "local" stocks, or stocks with which they have more familiarity. In our setting, local investors may feel that they have greater prior precision about local companies, and thus they require less of a risk premium to hold such assets.[15]

What of firms who are at the other end of the spectrum, the firms who are entering the market for the first time? Certainly, the effect in equation (20) would suggest that the low prior precision on those firms would increase the cost they face in raising capital.[16] But these firms face other problems as well. In particular, it may be that for some firms, there is little public information available. In our analysis thus far, we have considered the cross-sectional differences that arise when firms have the same total amount of information, but the composition of information between public and private sources may differ. For new firms, however, it seems likely that there is less information overall, and what information exists is more likely to be private. How, then, does this affect the cost of capital?

One way to address this question is to consider the role of private information in isolation. That is, if there were no public information, would a firm be better off having some private information or no private information? Proposition 4 demonstrates that having information is always better than not having information.

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[15] Such geographical preferences may also lend insight into the well-known phenomena of home country bias. If domestic investors feel that they have greater prior precision for domestic stocks, then they may view domestic stocks as having a lower risk-return trade-off than they face with foreign stocks. See Brennan and Cao [1997] for an analysis of this effect.

[16] Since most new firms are also small firms, this effect would also be consistent with the empirical regularity that small firms returns are higher than would be predicted by a standard CAPM. Indeed, Ibbotson Associates (2000; page 141) notes that "Based on historical return data on NYSE decile portfolios, the smaller deciles have had returns that are not fully explained by the CAPM. This return in excess of CAPM grows larger as one moves from

Proposition 4: Suppose $a_k = 1$. Then, for any firm k,

$$\frac{\partial E\left(\tilde{v}_k - p_k\right)}{\partial I_k} < 0$$

Proof: See Appendix

The result in Proposition 4 may appear paradoxical; in a world with no public information, having some private information will lower a firm's cost of capital relative to what it would be if there were no private information. One might have conjectured that uninformed investors would prefer a stock with no informed traders, but this is not the case. This is because of the effect that information has on the asset's equilibrium price. With some traders informed, this price will be more informative, and this lowers the risk for the uninformed. Of course, the risk and thus the firm's cost of capital will be even lower if the information is public (this is our finding in Proposition 3). But given the choice of no information, or only private information, the firm's cost of capital is lower in a world in which someone knows something.

This distinction between the existence of information in general, and the distribution of private and public information in particular, provides a way to reconcile our findings with those of previous models of the effects of information on asset prices. In a model with one risky and one risk-less asset, Wang [1997] found that private information did not generally result in a risk premium for the risky asset. This finding is consistent with the intuition behind Proposition 4 in which some information is better than none. In our model with many assets and public and private information, we find that there is a risk premium, and that it varies within the cross-section of

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the largest companies to the smallest." Such an effect could be explained by the information issues we highlight here.

stocks. These cross-sectional effects arise because of the portfolio channel discussed earlier. In general, one would expect that both effects would be present to some extent, but in any multi-asset world these cross-sectional effects will be present.

This dichotomous role of information may also explain the impact of insider trading laws on a company's cost of capital. The Manne [1966] argument against insider trading prohibitions essentially viewed some information, even if it were private, as better than no information at all (again our Proposition 4 result). Bhattacharya and Dazouk [2000] in a comprehensive empirical study of 103 countries, however, estimate that the enforcement of insider trading prohibitions reduces the cost of capital by between 0.3% and 6.0%. Assuming that the effect of these laws is to turn at least some of the private information into public information then this effect is predicted by our model: reducing the risk of informed trading, and correspondingly increasing the amount of public information, reduces the risk premium uninformed traders demand to hold the stock.

Finally, we consider one other effect on the company's cost of capital. As shown in Proposition 2, the level of risk aversion enters into the determination of the risk premium. The risk aversion level is not stock specific, and so it is not within the purview of a company to influence it. However, it is straightforward to show that increases in the risk aversion parameter will directly increase the risk premium demanded by investors.

This has two implications for our analysis. First, if the level of risk aversion changes over time, then we might expect to find the dispersion of cross-sectional returns changing as well. This occurs because investors need even greater compensation to hold stocks with more private information when the risk aversion parameter increases, and conversely they need less compensation when it falls. Such changes in cross-sectional return dispersion seem consistent with actual asset price behavior. Second, if risk-aversion is time varying, then this may explain the cycles we observe in firms coming to market. It is well known that IPOs exhibit a "feast or famine" cycle, with firms typically clustering together in coming to market. Since IPO firms have greater private information and low information precision overall, our model would predict a high risk premium to induce investors to hold them. An increase in overall risk aversion will cause this premium to increase even more, thereby inducing some firms to wait for the "better market conditions" consistent with a lower risk aversion level.

5. Extensions and Generalizations

In this paper we use a simple rational expectations model to make our arguments. A natural concern is that the simplicity of the model, while useful for illustrative purposes, is chimerical; that in a fuller model, the results we are interested in would no longer hold. In this section, we address this issue by considering how some extensions and generalizations to the model affect our results.

A. Correlation structures

Our model assumes independence of values and signals across assets. This framework greatly simplifies calculating a rational expectations equilibrium, but it does so at the obvious cost of minimizing the benefits of diversification across stocks. Allowing correlated payoffs across stocks does not alter our results on the effects of private information on risk premia unless the values are perfectly correlated. In the absence of such perfect correlation, agents are unable to completely diversify away the information risk we analyze here. Certainly, agents in equilibrium will remove some overall risk by diversifying, and the greater the correlation across assets, the more they can reduce the portfolio's overall variance. But, as explained in Section 3, the absence of a single market portfolio (and the consonant inability of the uninformed to mimic the exact portfolio weights of the informed traders' portfolio) means that there will always be a divergence between the risk borne by uninformed and informed agents in holding asset k even in the optimally diversified portfolio. In equilibrium, this divergence requires compensation. The magnitude of this needed compensation is an empirical question.

B. Multi-period effects

The model here incorporates two-periods, today and tomorrow. Allowing for multiple periods complicates the analysis, but for at least some reasonable specifications does not affect our results. For example, it is fairly straightforward to demonstrate that if new information arrives every period and information is independent across periods, then our results are unchanged. In this world, agents essentially solve the same decision problem period after period that they solve here, and so the resulting equilibrium remains the same.

This need not be true for every information structure. If private information once revealed now reduces permanently the private information over the **lifetime** of a stock, then one could get the paradoxical result that high information stocks are less risky to hold since the risk decreases over time faster than the risk of low information stocks. Again, while conceptually possible, we do not find this economically plausible. To the extent that information revealed today reduces information asymmetry tomorrow, then we would expect this to diminish, but not remove, the effects outlined here.

C. Interpretations of the Asset Structure

The asset structure we analyze results in some asset specific risks being less than completely diversifiable. Such an outcome also arises in asset pricing models in which factors such as HML (the book to market effect) or size are priced. What allows those risks and the information risk we derive here to be priced is simply that agents must be compensated for risk, and the asset structure alone is not sufficient to remove the risk.[17] While this incompleteness can arise naturally from the nature of the assets in the economy, it can also arise if some investors are constrained in the assets that they can hold. Such participation constraints motivate Merton's [1987] analysis of asset pricing with incomplete information about the set of available assets. This idea has since been explored extensively by several authors including Basak and Cuoco [1998] and Shapiro [2000]. A similar limited participation explanation is used to explain the results on home bias in asset portfolios (see, for example, Stulz [1981]). While we do not use a participation argument here, such a constraint could also result in information risks being priced.

6. Conclusions

We have developed an asset pricing model in which both public and private information affect asset returns. Because the return investors demand determines a firm's cost of equity capital, our analysis provides the linkage between a firm's information structure and its cost of capital. We have demonstrated that investors demand a higher return to hold stocks with greater private information. This higher return reflects the fact that private information increases the risk to uninformed investors of holding the stock because informed investors are better able to shift

[17] Fama and French [1992], for example, argue that HML and size proxy for firm risk sensitivities to factors such as distress risk.

their portfolio weights to incorporate new information. Private information this induces a new form of systematic risk, and in equilibrium investors require compensation for bearing this risk.

An important implication of our research is that firms can influence their cost of capital by affecting the precision and quantity of information available to investors. This can be accomplished by a firm's selection of its accounting standards, as well as through its corporate disclosure policies. Attracting an active analyst following for a company can also reduce a company's cost of capital, at least to the extent that analysts provide credible information about the company. Yet another way to influence its information structure is through the firm's choice of where to list their securities for trading. Because investors learn from prices, the microstructure of where a firm's securities trades can influence how well and how quickly new information is impounded in the stock price. These factors suggest that a firm's cost of capital is determined, at least partially, by corporate decisions unrelated to its product market decisions.

Our findings here raise a number of issues for further study. If, as our analysis suggests, the quality of information affects asset pricing, then how information is provided to the markets is clearly important. Recently, the SEC has considered allowing individual investors access to IPO electronic road shows, has proposed tighter restrictions on what companies can disclose privately to analysts, and has pondered whether internet investment chat rooms are positive or negative influences for stock prices. While addressing each of these topics is beyond our focus here, the framework we develop does provide a way to consider how particular market practices affect equilibrium asset pricing. Our results also raise interesting questions about security market design and the cost of capital. In particular, how transparency of trades and orders influence the informativeness of stock prices, or even how the speed of the trading system affects information flows to investors, seem important directions for future research.

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Appendix

Proof of Proposition 1

It is sufficient to show that there is a solution to the market clearing equation (13) of the form given in the statement of the proposition. Substituting into (13) for demand by informed and uninformed traders gives

$$m_k\left[\frac{r_k\bar{v}_k+g_k\sum_{i=1}^{I_k} s_{ki}-p_k\left(r_k+g_kI_k\right)}{d}\right]$$
$$+\left(1-m_k\right)\left[\frac{r_v\bar{v}_k+g_k\sum_{i=a_kI_k+1}^{I_k} s_{ki}+r_{qk}q_k-p_k\left(r_k+g_k\left(1-a_k\right)I_k+r_{qk}\right)}{d}\right]=x_k. \tag{21}$$

So,

$$p_k=\frac{r_k\bar{v}_k+g_k\sum_{i=a_kI_k+1}^{I_k} s_{ki}+m_kg_k\sum_{i=1}^{a_kI_k} s_{ki}+\left(1-m_k\right)r_{qk}q_k-dx_k}{r_k+g_k\left(1-a_k\right)I_k+m_ka_kg_kI_k+\left(1-m_k\right)r_{qk}}. \tag{22}$$

Both q_k and r_{qk} involve coefficients from the conjectured price equation (7) in the form of $\left(\frac{d}{b}\right)$. Substituting in for q_k in (22) from (8) and simplifying shows that

$$\frac{d}{b}=\frac{d}{m_kg_k}.$$

So by (8) and (9)

$$r_{qk} = \left[\left(m_k g_k a_k I_k \right)^{-2} h_k^{-1} d^2 + \left(a_k I_k g_k \right)^{-1} \right]^{-1}, \tag{23}$$

and

$$q_k = \frac{\sum_{i=1}^{a_k I_k} s_{ki}}{a_k I_k} - \left(\frac{s}{m_k g_k a_k I_k} \right) \left(x_k - \bar{x}_k \right). \tag{24}$$

Substituting (23) and (24) into (22) and solving for the coefficients yields the price equation and coefficients given in the proposition. This equation is of the conjectured form (7) so it is a rational expectations equilibrium.

Proof of Proposition 2

The expected return on stock k is, by Proposition 1,

$$E\left[v_k - p_k \right] = E\left[v_k - a\bar{v}_k - b \sum_{i=1}^{a_k I_k} s_{ki} - c \sum_{i=a_k I_k + 1}^{I_k} s_{ki} + dx_k - e\bar{x}_k \right].$$

The mean of each s_{ki} is $\bar{v}_k$ and the mean of x_k is $\bar{x}_k$, so

$$E\left[v_k - p_k \right] = \bar{v}_k \left[1 - a - b a_k I_k - c\left(1 - a_k \right) I_k \right] + \bar{x}_k \left(d - e \right).$$

Using the coefficients from Proposition 1, computation shows that $1 - a - b a_k I_k - c\left(1 - a_k\right) I_k = 0$ and that $d - e = d/C_k$. Thus

$$E\left[v_k - p_k\right] = \frac{\boldsymbol{d}\bar{x}_k}{\boldsymbol{r}_v + \left(1 - \boldsymbol{a}_k\right) I_k \boldsymbol{g}_k + \boldsymbol{m}_k \boldsymbol{a}_k I_k \boldsymbol{g}_k + \left(1 - \boldsymbol{m}_k\right) \boldsymbol{r}_{qk}}.$$

Proof of Proposition 3

Using the result of Proposition 2, calculation shows that

$$\frac{\partial E\left[v_k - p_k\right]}{\partial \boldsymbol{a}_k} = \frac{\boldsymbol{d}\bar{x}_k \left(1 - \boldsymbol{m}_k\right)}{\left(C_k\right)^2} \left[I_k \boldsymbol{g}_k - \frac{\partial \boldsymbol{r}_{qk}}{\partial \boldsymbol{a}_k} \right].$$

Then

$$\frac{\partial \boldsymbol{r}_{qk}}{\partial \boldsymbol{a}_k} = \frac{I_k \boldsymbol{g}_k \left(2\boldsymbol{a}_k^{-1} \boldsymbol{m}_k^{-2} I_k^{-1} \boldsymbol{g}_k^{-1} \boldsymbol{h}_k^{-1} \boldsymbol{d}^2 + 1 \right)}{\left(\boldsymbol{m}_k^{-2} \boldsymbol{a}_k^{-1} I_k^{-1} \boldsymbol{g}_k^{-1} \boldsymbol{h}_k^{-1} \boldsymbol{d}^2 + 1 \right)^2}.$$

So

$$I_k \boldsymbol{g}_k - \frac{\partial \boldsymbol{r}_{qk}}{\partial \boldsymbol{a}_k} = I_k \boldsymbol{g}_k \left(1 + \boldsymbol{a}_k I_k \boldsymbol{h}_k \boldsymbol{m}_k^2 \boldsymbol{g}_k \boldsymbol{d}^{-2} \right)^{-2}.$$

Thus

$$\frac{\partial E\left[v_k - p_k\right]}{\partial \boldsymbol{a}_k} = \frac{\boldsymbol{d}\bar{x}_k \left(1 - \boldsymbol{m}_k\right) I_k \boldsymbol{g}_k}{\left(C_k\right)^2 \left(1 + \boldsymbol{a}_k I_k \boldsymbol{h}_k \boldsymbol{m}_k^2 \boldsymbol{g}_k \boldsymbol{d}^{-2} \right)^{-2}} > 0.$$

Proof of Proposition 4

Using the form of the expected return from Proposition 2, calculation shows that at $\boldsymbol{a}_k = 1$,

$$\frac{\partial E\left[v_k - p_k\right]}{\partial I_k} = -\frac{\boldsymbol{d}\bar{x}_k \left(1 - \boldsymbol{m}_k\right)}{\left(C_k\right)^2} \left(\boldsymbol{m}_k \boldsymbol{a}_k \boldsymbol{g}_k + \left(1 - \boldsymbol{m}_k\right) \frac{\partial \boldsymbol{r}_{qk}}{\partial I_k} \right)$$

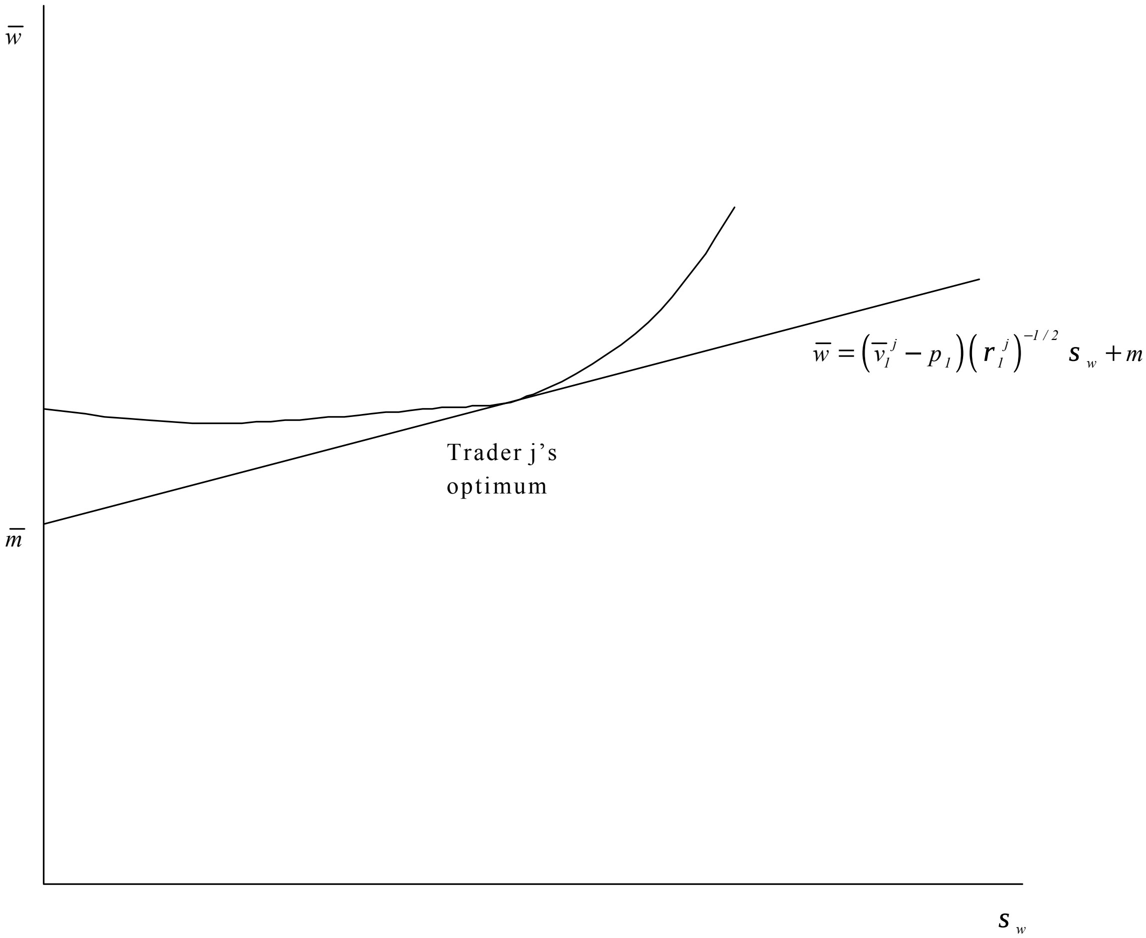
and

$$\frac{\partial \boldsymbol{r}_{qk}}{\partial I_k} = \frac{2I_k^{-3}\left(m_k g_k \boldsymbol{a}_k\right)^{-2} \boldsymbol{h}_k^{-1}\boldsymbol{d}^2 + I_k^{-2}\left(\boldsymbol{a}_k g_k\right)^{-1}}{\left(m_k g_k \boldsymbol{a}_k I_k\right)^{-2} \boldsymbol{h}_k^{-1}\boldsymbol{d}^2 + \left(\boldsymbol{a}_k I_k g_k\right)^{-1}} > 0.$$

So

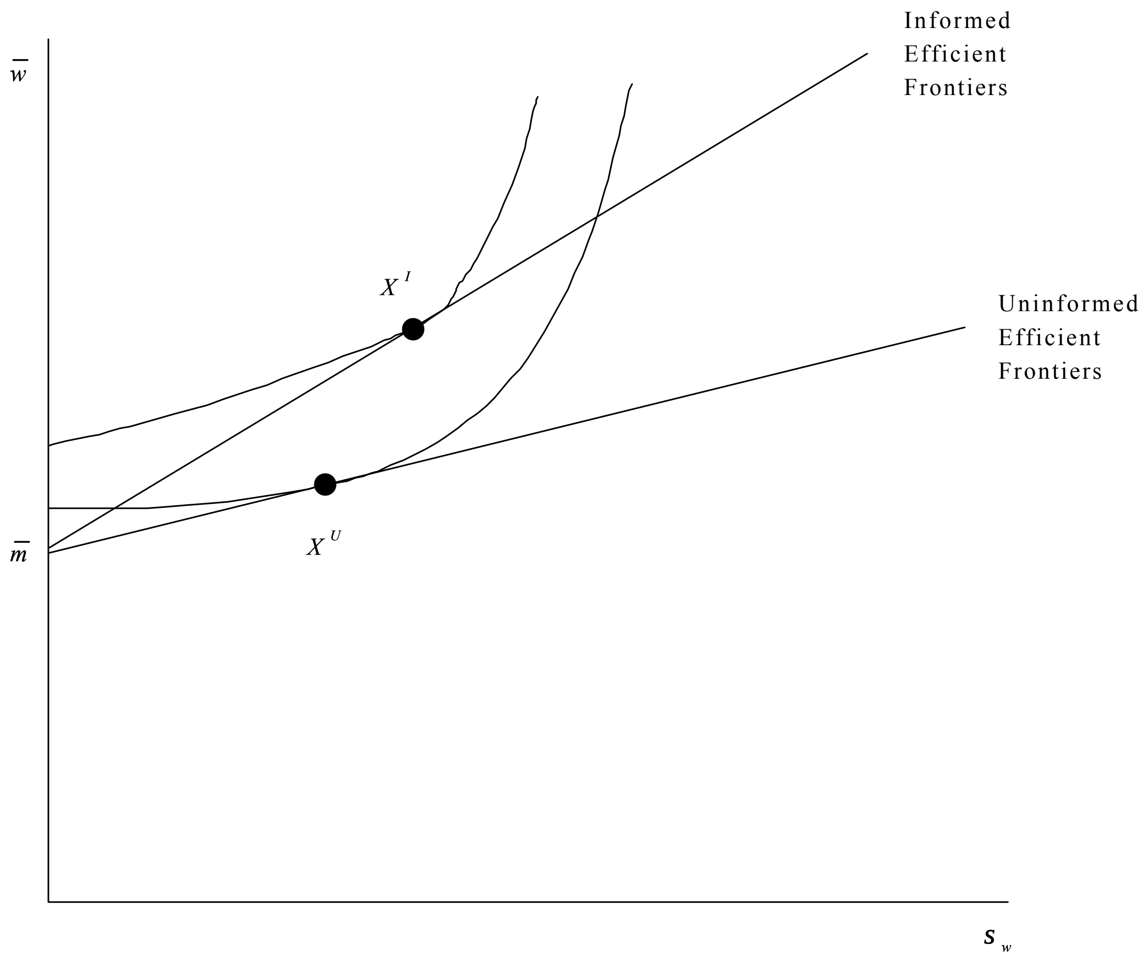
$$\frac{\partial E\left[v_k - p_k\right]}{\partial I_k} < 0.$$

Figure 1
The Efficient Frontier for Trader j



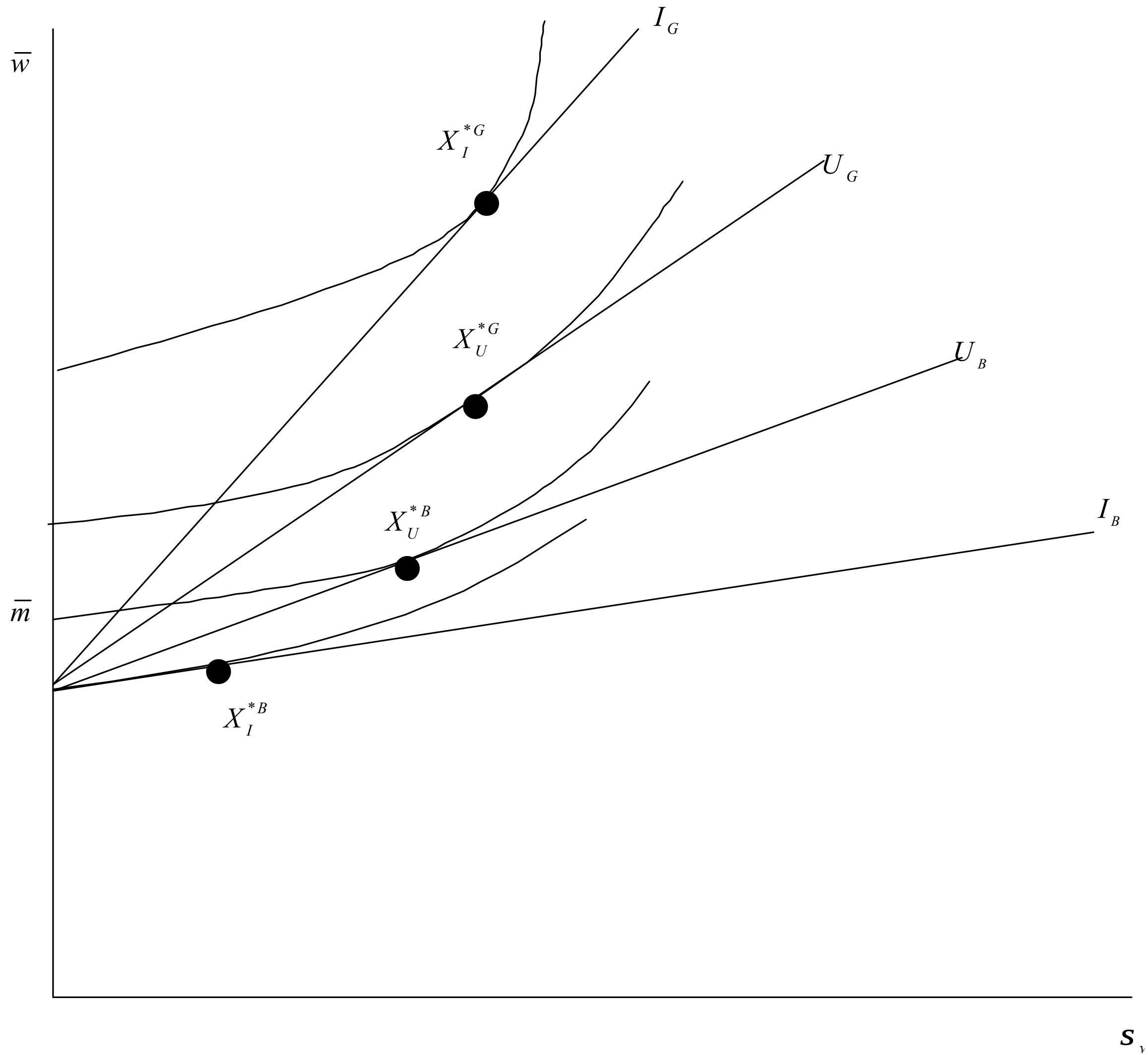
This figure shows the risk-return trade-off confronting an investor j. The perceived efficient frontier is linear, with a slope determined the trader's perception of mean returns and standard deviations for assets. For an economy with one risky asset (and one riskless asset) the slope of this frontier according to trader j is $(\bar{v}_1^j - p_1)(r_1^j)^{-1/2}$.

Figure 2
Efficient Frontiers for Informed and Uninformed Traders



This graph shows that the efficient frontier is different for informed and uninformed traders. The frontier for the informed trader is above that of the uninformed trader because the informed trader knows more about the asset, and so faces a lower risk-return tradeoff. X^I and X^U denote the average portfolio choices of informed and uninformed traders. The uninformed trader's optimal holding is less than that of the informed trader because the uninformed trader faces greater risk in holding the asset.

Figure 3
The Effect of Good and Bad News on Portfolios of Informed and Uninformed



This figure shows how the efficient frontiers change with respect to good and bad news for informed and uninformed traders. These efficient frontiers are given by I_G and I_B (U_G and U_B) for informed (uninformed) traders given Good news and Bad news, respectively. Informed traders' beliefs about mean return are more responsive to signals than are uninformed traders' beliefs. So when there is good news, the informed hold even more of the risky asset, and when there is bad news their holdings are reduced by more than are the uninformed traders' holdings. If the news is bad enough, the informed hold less of the risky asset than do the uninformed.